The Challenge of Computational Mechanics

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The Challenge of Computational Mechanics

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The honour of being today the recipient of the Gauss medal I take not only as a personal appreciation for which I am grateful, but as recognition of the importance of the subject of Computational Mechanics in modern engineering.

Since the foundation of mechanics have been laid by Sir Isaac Newton and the formulation of continuum mechanics and thermodynamics at the start of the nineteenth century, the Engineer has been provided with the basis by which he could assess the performance of his artefacts – be they dams and bridges of the civil engineer, combustion engine of the mechanical engineer or flying machines and satellites of this century's aerospace engineering.

However theoretical mechanics provided only the underlying science – and from that point to the quantitative prediction needed in design a large step remained.

In the words of von Karman,
Science studies what is –

– Engineering creates what never has been.

– In this process of 'creation', though imagination plays an essential role (which will never be usurped by computers), a detailed quantitative prediction is essential.

Such prediction is the essence of engineering science – and at all stages involves 'computation'. In the early part of this century the slide rule has been the symbol of the engineer as much as the drawing board. However many problems though understood well physically remained outside the engineers grasp through his inability to solve the mathematical equations involved. To overcome this, painstaking model and prototype experiments – with a high failure rate – had to be accepted as the norm.

Today the situation is drastically altered. The power of the computer, which has grown exponentially since the fifties, permits the numerical solution of very complex problems. The early work of such pioneers as Richardson (1910) also first realized that numerical solution of elasticity problems is feasible and Southwell (1940) who through relaxation methods made such solutions more widely acceptable formed the base on which modern numerical methods coupled with computational power make almost all problems tractable.

The finite element method – which was unthinkable as a practical procedure before the computer, and its special cases of finite difference, boundary solution methods, spectral approximations, etc. provide today a tool which, in principle at least, opens all doors.

The solution of complex, realistic behaviour through computational means forms the basis of the subject which today we term *Computational Mechanics* (or if we accept

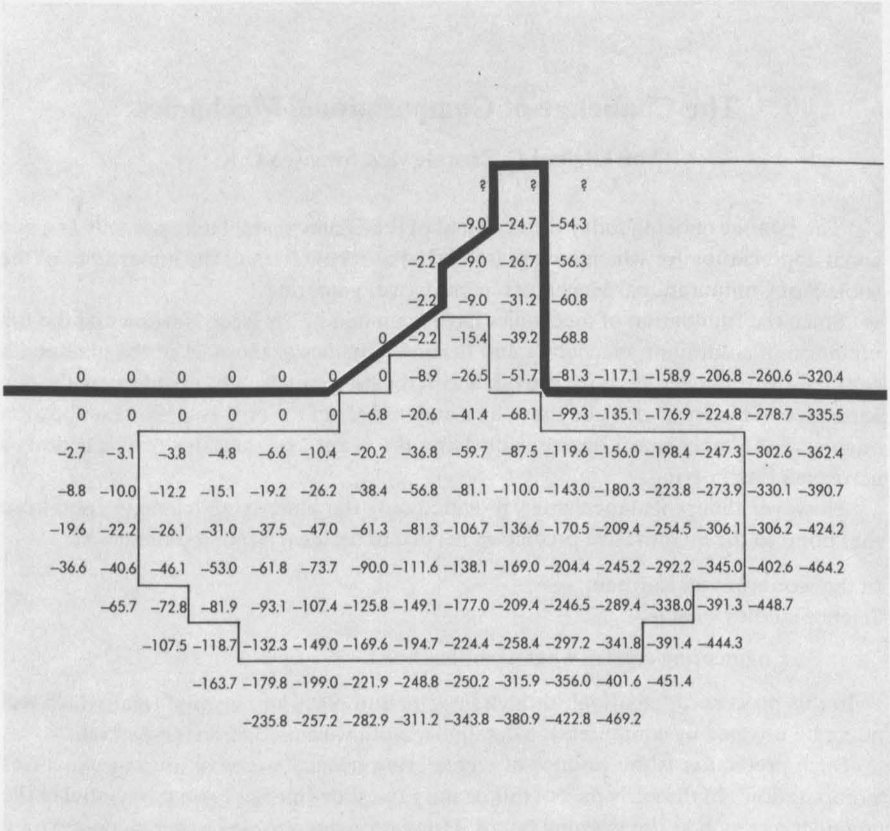


Fig. 1:
The first numerical approximation analysis. The Aswan Dam – L. F. Richardson.
(Numbers refer to the values of the Airy stress function).

that the possibilities offered can be extended beyond the field which clasically are associated with mechanics of *Computational Moedelling* phenomena).

Figures 1 through 5 illustrate some of the early as well as more recent computer predictions of engineering situations. Clearly much has been achieved and commercial codes available to all (at a price) allow the engineer to solve readily many situations which were, before the advent of the computer, intractable. CAD (Computer Aided Design) incorporates many of such 'solutions' and in some industries this is used daily.

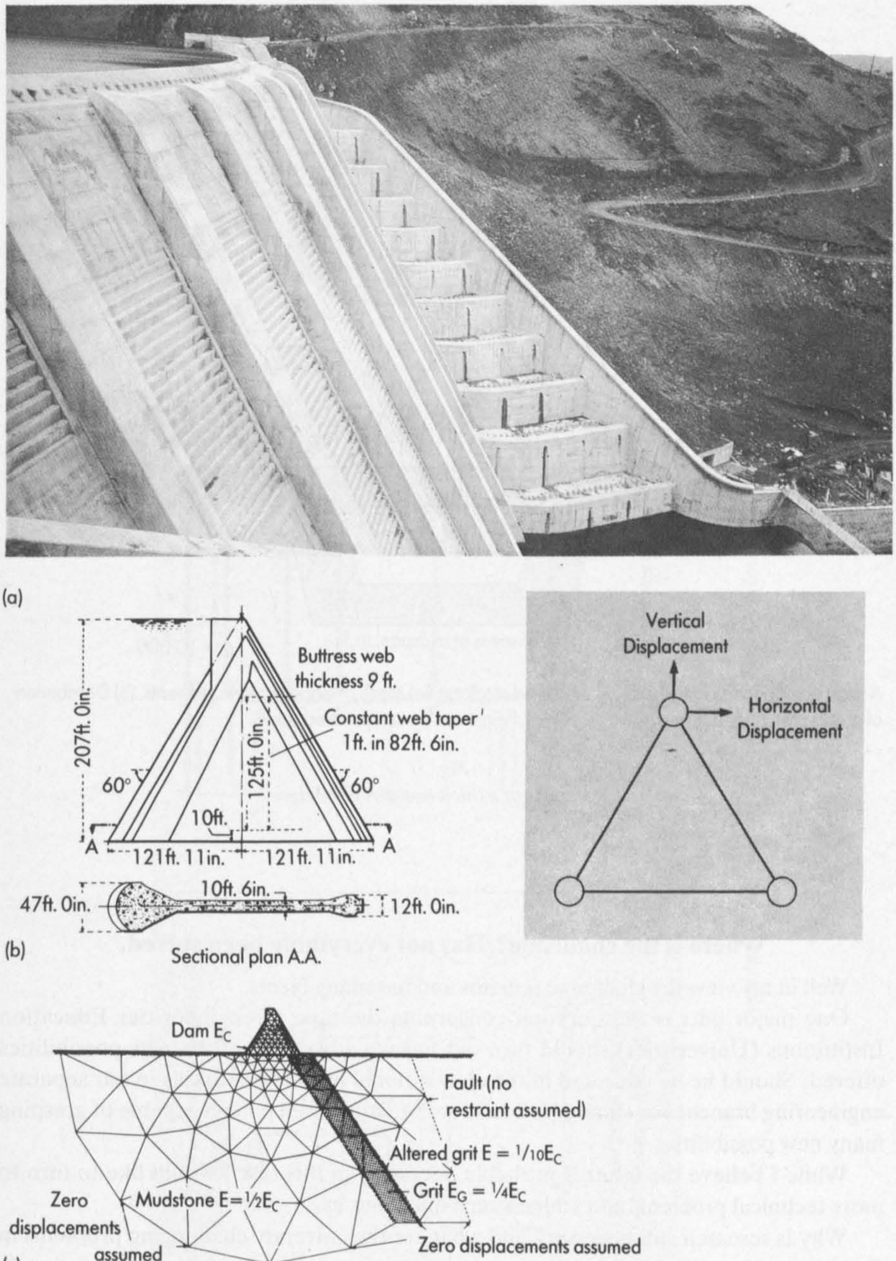
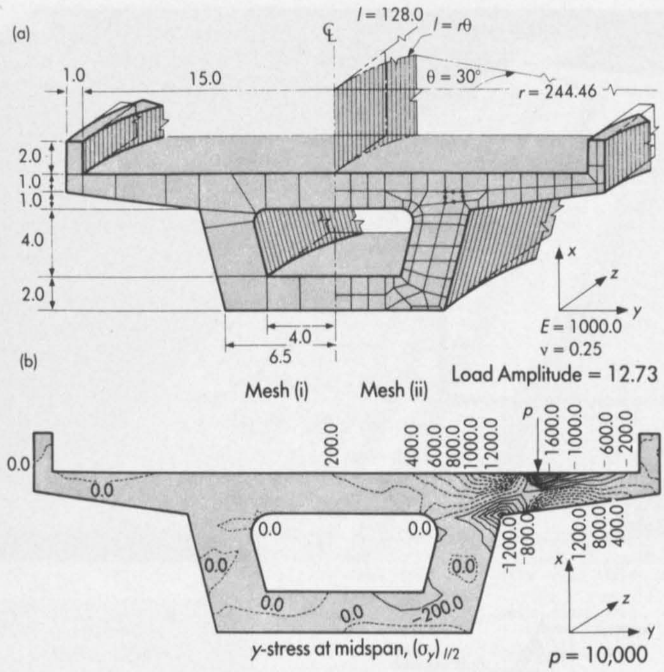


Fig. 2:
First finite element analysis in design process. Clywedog Dam, Wales 1964.



A thick box bridge prism of straight or curved platform. (a) Mesh of isoparametric elements. (b) Distribution of σ_y stress on midspan; computer stress plot. Point load on cantilevered span.

Fig. 3:
Analysis of a thick box girder bridge.

Where is the challenge? Has not everything been solved.

Well in my view the challenge remains and has many facets.

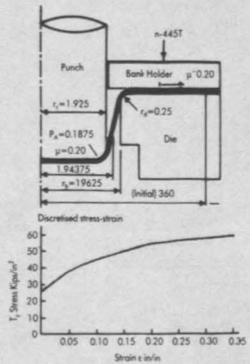
One major part is the question concerning the type of engineer our Education Institutions (Universities) should turn out to take advantage of the new possibilities offered. Should he be educated in the conventional professional skills in our separate engineering branches or should he be a more broadly based person capable of grasping many new possibilities.

While I believe the latter is probably necessary in this talk I would like to turn to more technical problems and address such questions as:

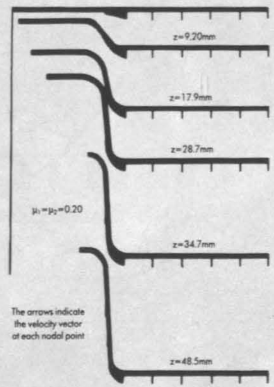
Why is research still necessary and what are the currently challenging problems in the computational field?

How the impact of Computational Mechanics affects other aspects of mechanics and engineering and what are the outstanding problems of this interface?

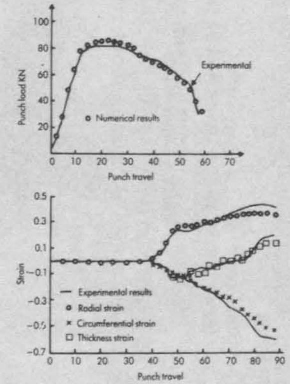
Geometry and finite element discretisation of the blank using linear elements



Flat nosed punch. Geometry and discretised stress–strain curve – Alloy 84



Flat bottom punch. Deformation at different stages



Flat nosed punch. Strain distributions for punch travel = 60mm

Fig. 4:
Analysis of beer can forming process. Finite element solution and experiment.

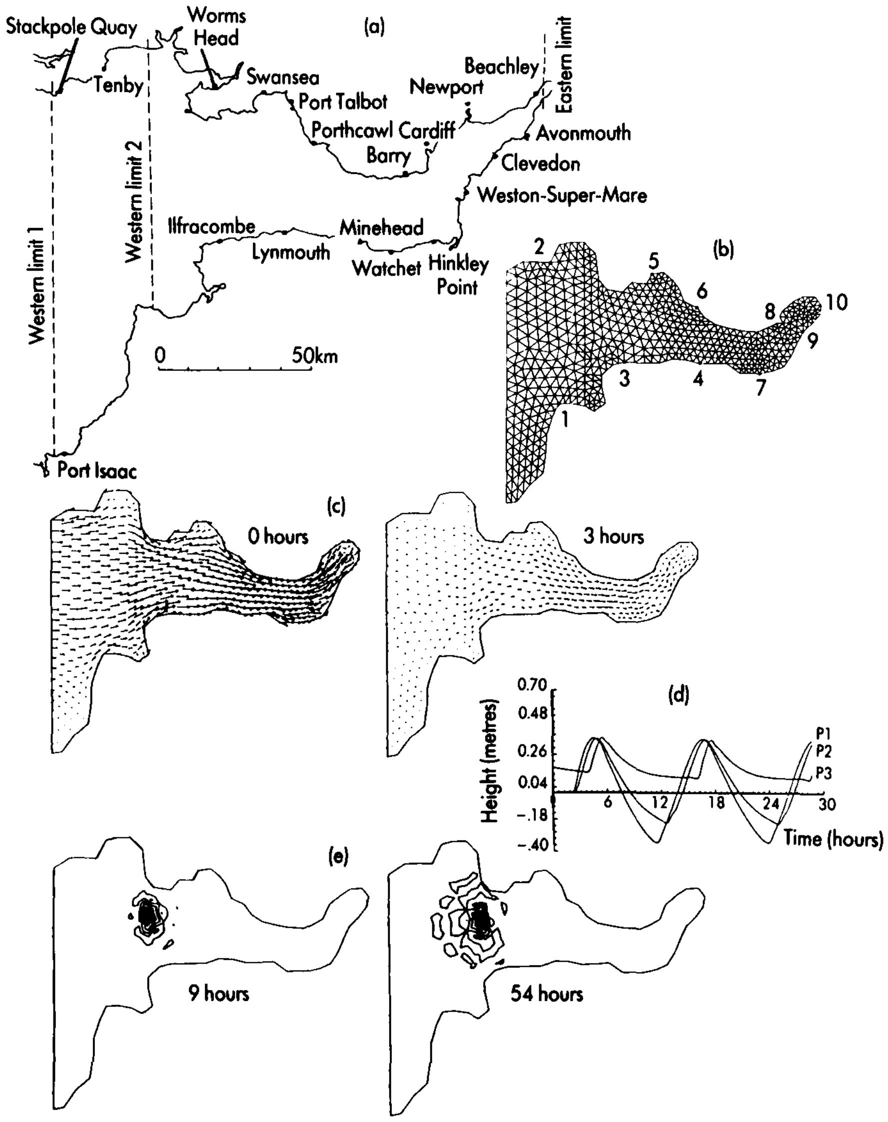


Fig. 5:
Tidal Area Solution for the Severn Estuary.
(a) Severn estuary and the Bristol Channel, the problem area. – (b) Finite element mesh. – (c) Velocity distribution in the tidal cycle. – (d) The River Severn and bore development. – (e) Dispersion of pollutant.

Why is research still necessary and the present problems of challenge?

While in principle the numerical methodologies available today should be capable of answering many points with sufficient computer power available, continued research is necessary to

(a) Increase the efficiency of the approximation and solution procedures to deal with constantly expanding demands.
(b) To ensure that the extent of approximation is understood by the user – and if required to improve this approximation economically to a specified standard.
(c) To derive numerical procedures for problems in which currently available methods fail or are inefficient.

To illustrate the importance of above I shall show how only recently breakthroughs have been achieved in the last two areas.

The first of these is the question of error estimation and adaptivity in finite element computation. Here it is only in the last decade that involvement of the mathematical

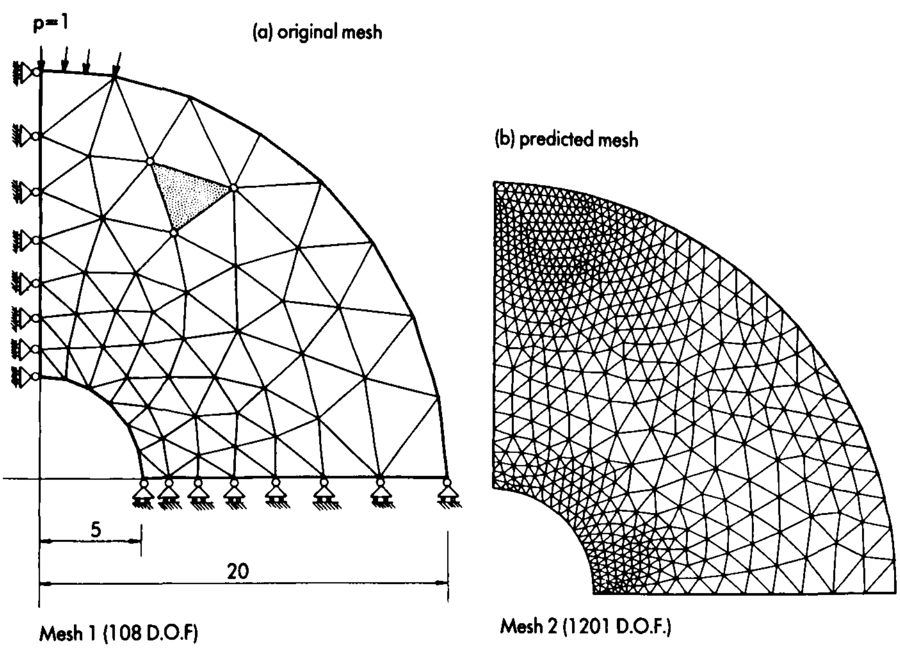


Fig. 6:
An adaptive refinement process for an elastic analysis using simple linear triangle elements.
(a) Originally designed mesh error 34% (108 degrees of freedom). –
(b) Mesh designed to reduce error to 5% actual error achieved 8% (1201 degress of freedom).
All meshes are automatically generated and error estimaters used.

fraternity (Babuska et al 1976) has introduced the possibility of constructing error estimates after a numerical solution has been accomplished. The need for such estimates is obvious as frequently the solution given by the computer, while appearing reasonable, may be wildly inaccurate (and perhaps some of our engineering failures may be attributable to this cause).

Despite the theoretical work available only very few commercially available codes have any facilities available for such error estimation – mainly due to the relatively high cost involved in the evalution of their early forms.

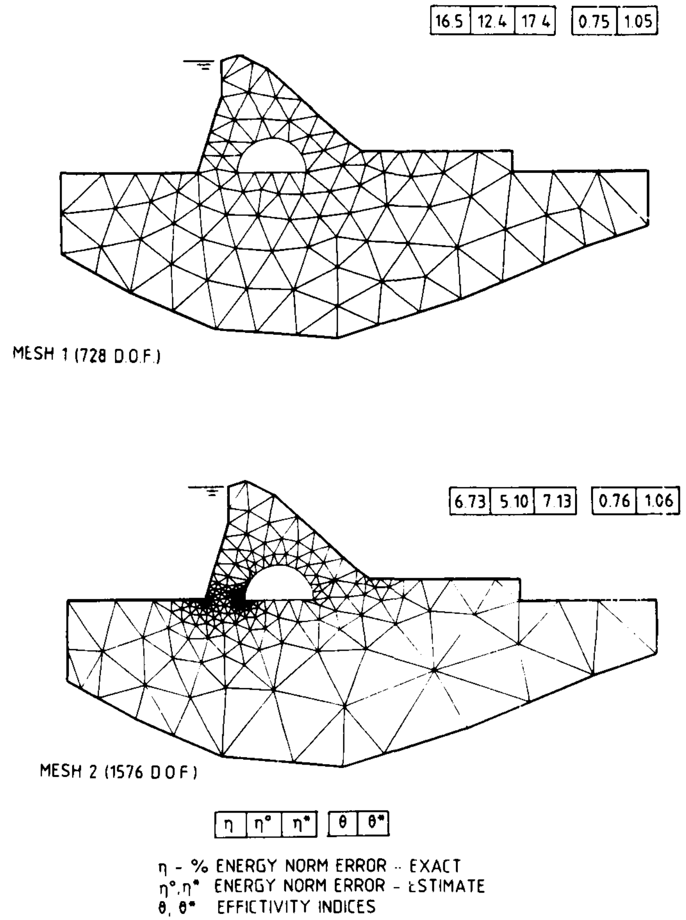


Fig. 7:
An adaptive analysis of a dam using quadratic triangles.
(a) Original mesh 728 DOF. error 16.5%. – (b) Mesh refined to achieve 5% error 1576 DOF. error 6.7%.

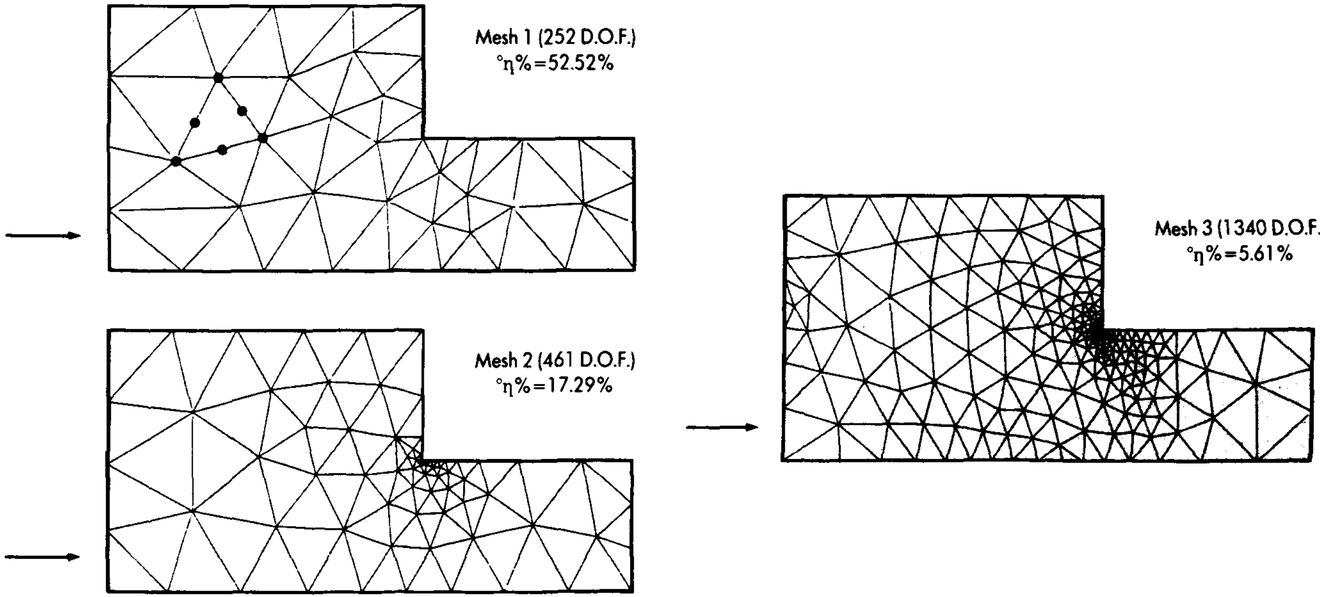


Fig. 8:
METAL EXTRUSION PROBLEM,
Progressive mesh refinement aimed of achieving 15% and 5% errors respectively.

Today the situation is changing dramatically and inexpensive computation of error estimation is available which achieves a high degree of efficiency (Zienkiewicz and Zhu 1987). Not only can global estimates be readily achieved but errors computed at local (element) level can be used to guide the refinement required to reach a given accuracy level.

This poses immediately the very practical question of how can mesh refinement of a given density be best achieved and here once again research of a practical (development) nature is required. Despite many years of activity in the field such mesh generators are not easily available and were derived only recently (Morgan et al 1986) allowing triangular grids to be so constructed.

Figures 6 and 7 show how, by the use of error estimation and prediction of required mesh density, solution of given accuracy can be generated in the most common elasticity problems.

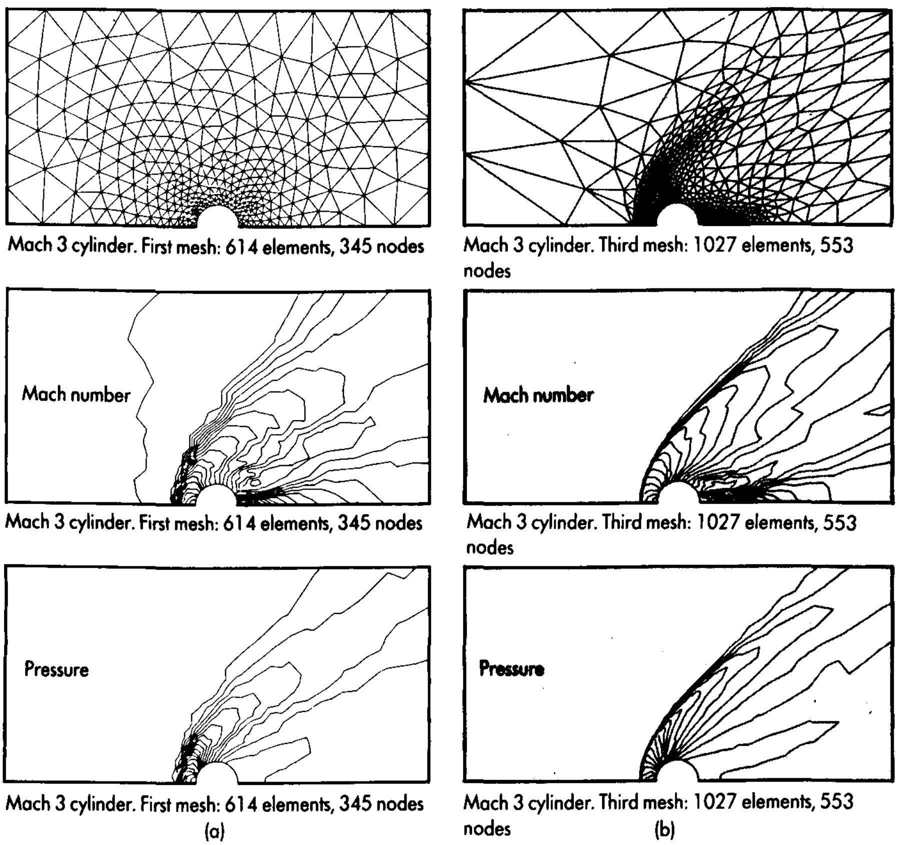


Fig. 9:
Mesh regeneration used adaptively to capture shock formation in high speed compressible gas flow around a cylinder.

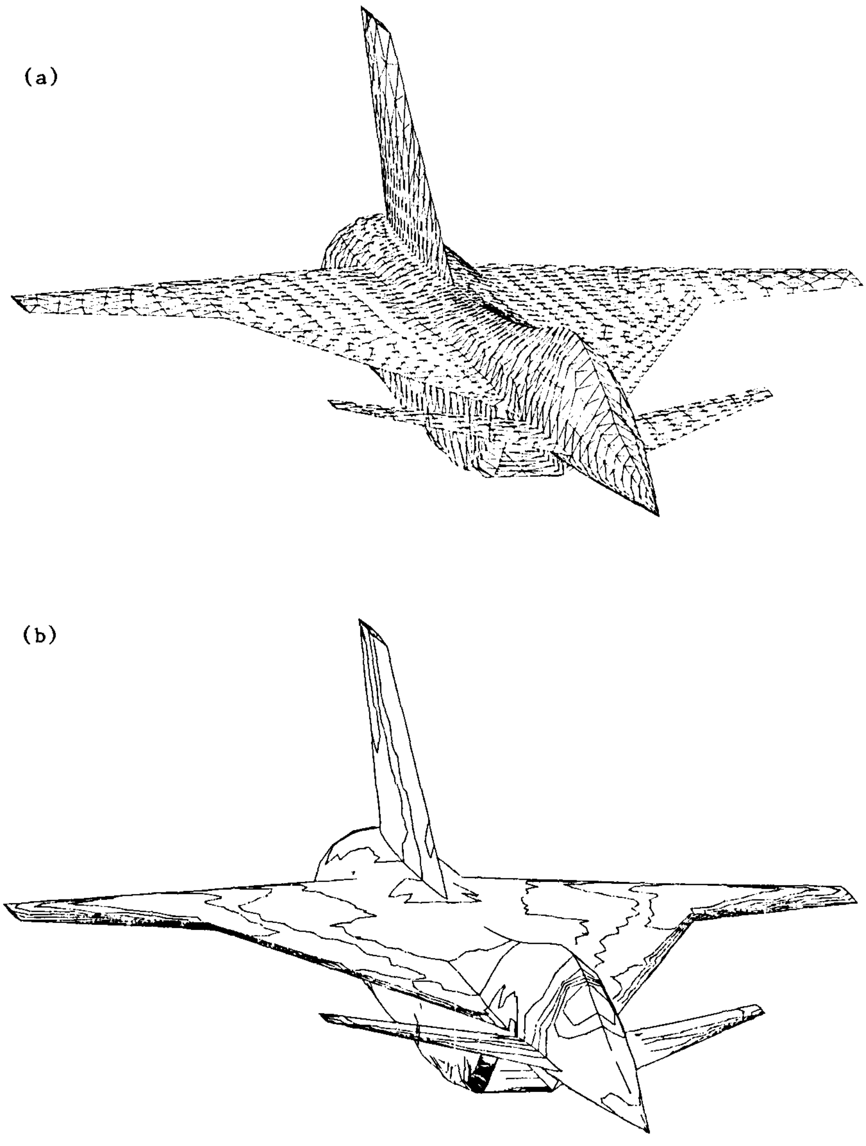


Fig. 10:
Cranked-winged experimental fighter configuration with engine intake. The finite element 3D mesh about the complete aircraft consists of 153.044 tetrahedral elements and 28.182 nodes. (a) shows the surface triangulation and (b) the pressure coefficient distribution on the surface for an inviscid flow simulation at Mach 2 with 3° angle of incidence. Computed by J. Peaire, J. Peiro and L. Formaggio from the Institute of Numerical Methods in Engineering, Swansea.

Clearly extension of such practical procedures into other areas of computation is necessary and much current work is in progress. Figure 8 illustrates an application in a non-linear extrusion process.

The second of the examples in which only recent progress has been achieved is the area of fluid mechanics where the efficient and stable solution of convective terms is still a subject of current research.

In this area finite difference methods have predominated until the early eighties but tied to 'structured' grids their ability to tackle high speed flow and shock formation was very limited. In the last five years considerable progress has been achieved in understanding of the mathematics involved and the introduction of finite element methods into this area allows detailed studies of shock formations and other phenomena. The first solutions of flow around complete aircraft are now being attempted.

The major breakthroughs were achieved by my own group (Loehner, Morgan, Zienkiewicz, Peraire 1984–1987, and it gives me much pleasure to mention that the first of the students involved in this work came from the University of Braunschweig (showing how much can be achieved by free international co-operation).

The procedures developed involve not only novel formulations but utilize both error estimation and adaptivity to ensure efficient shock capturing.

Figures 9 and 10 show some of the applications to such problems in which both mesh 'enrichment' and mesh regeneration are used to achieve desired accuracy. Current work on three dimensional extension of the procedures promise to allow full studies of the proposed European space shuttle – the Hermes, and indeed of the British HOTOL to be undertaken.

The methodologies developed in one field allow rapid transfer of areas and in the above case permitted simultaneous solution of many coastal engineering problems (Peraire, Zienkiewicz 1986).

Impact of computational mechanics on other areas of engineering mechanics

The possibilities of computational solution of hitherto insoluble problems stimulate parallel research which previously was at best of 'academic' interest.

A case in point is the area of soil dynamics where the possibility of solving the coupled soil-pore fluid interaction led to a large effort in identifying constitutive laws of soil behaviour.

Here problems of soil liquefaction encountered in earthquake response and leading to many catastrophies have at last become amenable to quantitative solution. Such solutions have to be verified by detailed experiment and in Fig. 11 we show some results of recent collaboration work between Japan, Cambridge University and INME Swansea in which computational predictions are compared against a scale model tested in a centrifuge. The detailed comparison shows how much can be achieved by computation coupled with suitable mechanics modelling.

This example is typical of many situations in which the numerical, computation solution stimulates detailed and new studies of classical mechanics.

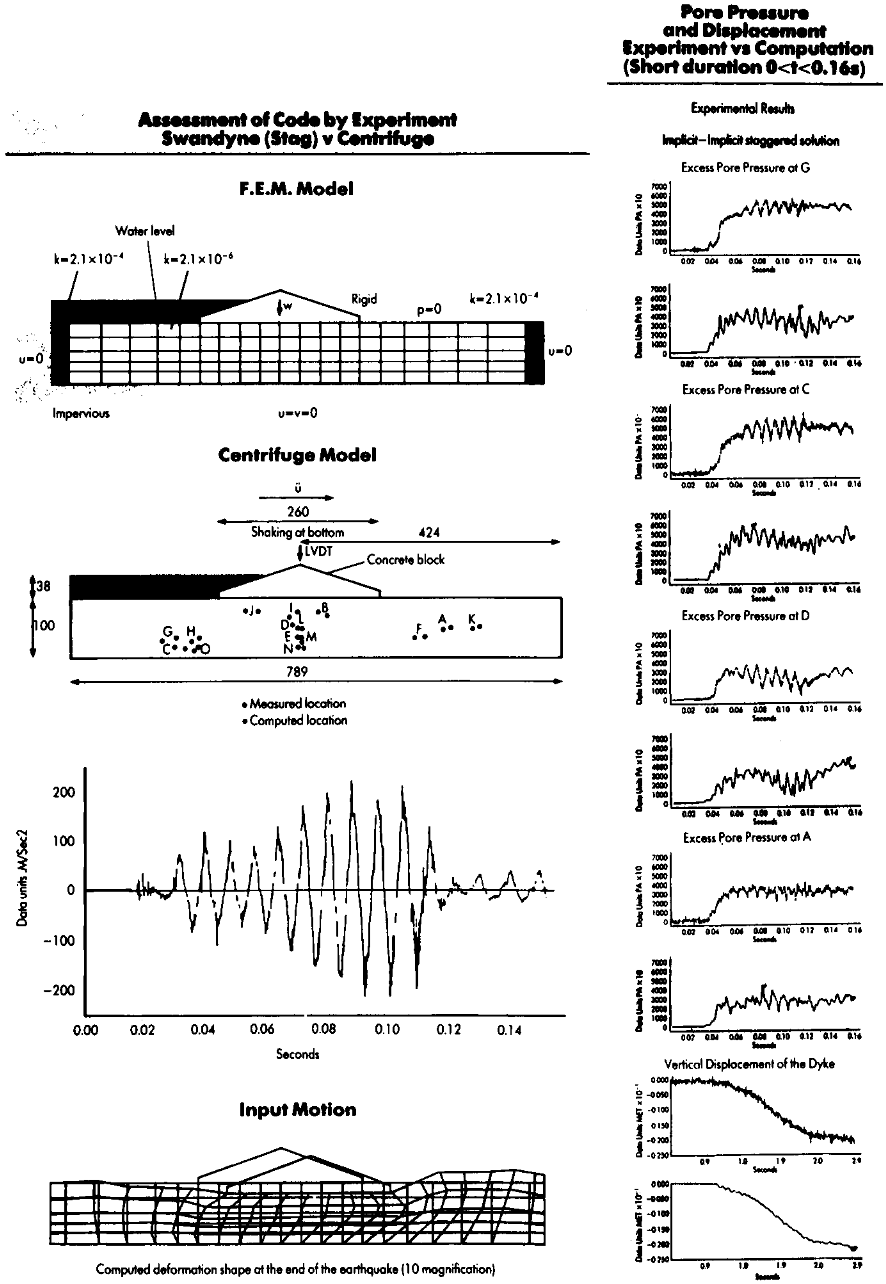


Fig. 11:

Centrifuge model of an earthquake soil liquefaction problem and corresponding finite element analysis results (Cambridge centrifuge and DIANA-SWANDYNE programm INME Swansea 1986).

Concluding remarks

The preceding has indicated by example the importance which should be attached to the subject of Computational Mechanics and some features of its impact on classical mechanics and engineering. Much more can be said about the subject which today and in the future will become more and more central to the whole of Engineering.

The 'buzz' words of *Reliability and Robustness* of computation are today much in evidence and though I have indicated some aspects connected with above much more needs to be done in the future.

An area on which I have not touched concerns the matter of Engineering Design Criteria, which today have to be considerably adjusted to take consistently into account the more precise knowledge of behaviour. Indeed in this area such topics as the "bracketing" of unknown parameters and the new area of *stochastic analysis* provides much incentive for future work.

The DREAM of many in which complete analysis can be carried out optimally and with the minimum of human intervention is achievable in some instances but its very possibility poses many dangers. It is in my view essential that only through fully professional understanding of all processes involved will true progress be made in the field of Computational Mechanics and Engineering.

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